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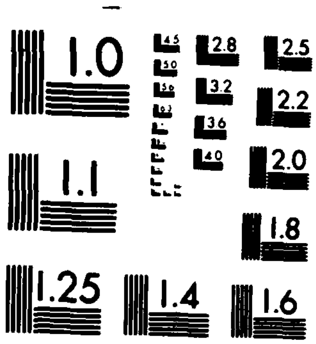
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PERFORMANCE OF A VARIABLE-CONSTRAINT-LENGTH VITERBI DECODING ALGORITHM

JK Tamaki

August 1982

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NAVAL OCEAN SYSTEMS CENTER
San Diego, California 92152

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The high-frequency channel is investigated and analysis undertaken of the computational complexity of the Viterbi decoding algorithm for the time-variant fading channel. Since the effective constraint length may expand or contract, it is shown that one can expand or contract the state trellis diagram of the Viterbi algorithm accordingly. It is also shown that certain channels have output sequences that cannot be uniquely decoded. Examples of such catastrophic codes are discussed.

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SUMMARY

OBJECTIVE

Document demonstrates

~~To show~~ that the computational complexity of the Viterbi decoding algorithm can be reduced for Maximum Likelihood Sequence Estimation on time-variant, fading channels when the channel constraint length is reduced.

~~RESULTS~~

An algorithm is developed that reduces the computational complexity of the Viterbi decoding algorithm. For time-variant, fading channels, the effective constraint length may expand or contract, and it is shown that we can expand or contract the state trellis diagram of the Viterbi algorithm accordingly. The derived algorithm requires knowledge of the channel's intersymbol interference patterns. These results apply not only to the hf channel, but to any channel characterized by convolutional encoding. In addition, channels that demonstrate catastrophic behavior are found and discussed.

CONTENTS

I. INTRODUCTION

Communication signal transmissions in the high frequency (hf) band (2-32 MHz) propagate thousands of miles by reflecting off the ionosphere. This feature has long been exploited to provide long-range communications, such as ship to ship and ship to shore. However, many propagation paths (reflections) are possible, and messages are frequently garbled due to the delayed echoes that arrive at the receiver. This problem causes severe degradation to digital communication links. The delayed echoes cause intersymbol interference, which results in very high symbol error rates.

One way to approach this problem is to treat the channel as a waveform encoder (ref 1). The receiver can then recover the transmitted data by decoding the received waveform. This technique is called Maximum Likelihood Sequence Estimation (MLSE) (ref 1-6) and uses the Viterbi algorithm (ref 7) to decode the received waveform. In the general problem of intersymbol interference on fading, time-variant channels, we will examine what effect fading will have on the complexity of the Viterbi algorithm.

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1. G. David Forney, "Maximum Likelihood Sequence Estimation of Digital Sequence in the Presence of Intersymbol Interferences," <u>IEEE Transactions on Information Theory</u>, Vol II-18, No. 3, May 1972, pages 363-378.
2. L. E. Hoff and A. R. King, "Skywave Communication Techniques," NOSC TR 709 30 Mar 1981.
3. L. E. Hoff, R. L. Merk and S. Norvell, "Maximum Likelihood Sequence Estimation for Unknown, Dispersive, and Time Variant Communication Channels," NOSC TR 727, 30 Sept 1981.
4. G. Ungerboeck, "Linear Receiver and Maximum-Likelihood Sequence Receiver for Synchronous Data Signals," <u>IEEE International Communications Conference Proceedings</u>, June 1973.
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6. A. J. Viterbi, "Convolutional Codes and Their Performance in Communication Systems," <u>IEEE Transactions on Communications Technology</u>, Vol-Com-19, No. 5, Oct 1971.
7. A. J. Viterbi, and J. K. Omura, <u>Principles of Digital Communication and Coding</u>, McGraw-Hill, New York, 1979.

As we model the multipath channel as a convolutional encoder, we first consider, in Sections II and III, a standard convolutional code sent over an additive white Gaussian noise (AWGN) channel using Viterbi (maximum likelihood) decoding. If the taps of the generator polynomials suddenly change at time t_o so that the constraint length does not increase, it should not affect the way in which the Viterbi algorithm has decoded prior to that time and should also not affect the metrics. Therefore, if the new code is known, the only change necessary in the Viterbi algorithm would be to update the code words generated by the channel in the trellis diagram. The concept of fading in and out affects the taps of the generator polynomials, and hence the constraint length may contract or expand. Since the Viterbi algorithm's complexity is exponential in the constraint length, we will show that while the channel is operating at the smaller constraint length, we can take advantage of this in the Viterbi algorithm without any loss of information concerning the metrics or survivors when the constraint length expands back. We will focus attention upon the case in which the taps fading to zero correspond to the higher degrees of the generator polynomials. Because the larger degrees correspond to the echoes, it is more likely that an echo fades to zero, as opposed to the first strong received signal fading to zero. It should be noted, however, that in the event the first signal fades to zero, one can always still decode using the larger constraint length even though the effective constraint length has dropped.

Let us assume that at time t_o the constraint length decreases from K to K'. We will show that if we identify as a group all states with the same initial K'-1 bits from time t_o (the <u>initial</u> K'-1 bits because we are assuming that the degree of the generator polynomials decreases from K-1 to K'-1), and if we define the metric of the group to be the best metric of all states in the same group, then maximum likelihood decoding on the reduced trellis decodes in the same fashion as would the larger trellis. This all assumes that $L \geq K-K'$, where L is the number of input bits processed at the smaller constraint length. (In all practical applications, we will have $L \gg K-K'$.) In the event that $L < K-K'$, we can substitute K-1-L for K'-1 above. The reduced trellis behaves exactly as the standard trellis diagram for a Viterbi decoder of constraint length K'-1.

When the channel expands to a larger constraint length, it is necessary for the Viterbi decoding algorithm to be given a warning of (K-K') input bits to prepare for the larger constraint length. This presents little difficulty as there is normally a delay due to the implementation of the algorithm that is much greater than the required (K-K') bit delay. However, if the delay for some reason does not naturally occur, one can always institute a mandatory delay of M bits, where M is an upper bound for the largest possible constraint length.

In section IV we set up a model for the hf channel and show that the results for convolutional codes extend to the time-variant hf channel.

When the echoes of the multipath channel are equally spaced in time and all of equal strength, the code produced by the channel is catastrophic. Other codes with taps of unequal strength are also shown to be catastrophic. In section V we examine this class of codes potentially generated by the hf channel.

Although the channel may give rise to a catastrophic code, given continuous fading over a randomly changing channel, the percentage of time these channels occur is very small. The echoes must all be exactly of equal strength and the path spacing must also be a multiple of the baud rate for a catastrophic code to arise. Furthermore, even when we have a catastrophic code, the error rate during a catastrophic burst could be as low as $\frac{2}{K-1}$, where K is the constraint length. There is much room for further investigation regarding the performance of these codes.

II. DEFINITIONS AND NOTATIONS

An attempt is made to keep the terminology for convolutional codes consistent with ref 6. (For further discussion, see ref 7 or 8.) Suppose we start with a convolutional code of constraint length K. All states will be a

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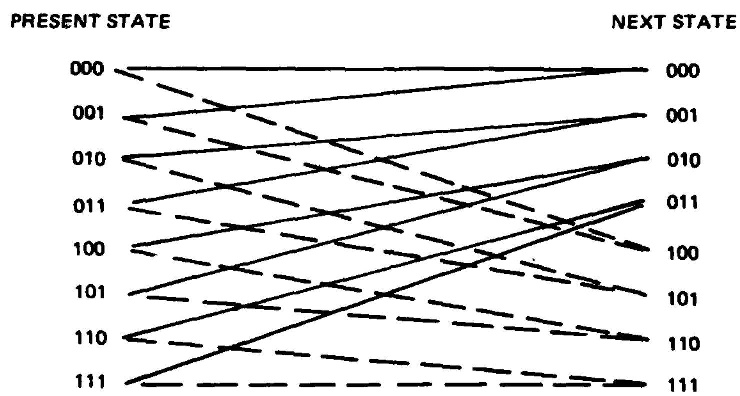
8. R. J. McEliece, "The Theory of Information and Coding," Encyclopedia of Mathematics and Its Applications, Vol. 3, edited by G. C. Rota, Addison-Wesley, 1977.

sequence of K-1 input symbols. If I is any input symbol, $S \xrightarrow{I} S'$ will denote the state transition from S to S' upon input of I, and we will call S' the <u>I-successor</u> of S, or write S' = I(S). (In other words, if $S = I_1 I_2 \ldots I_{K-1}$, then $S' = I I_1 I_2 \ldots I_{K-2}$.) Let pred (S') be the set of all states that map into S' via some input symbol I. For $m \leq K-1$, let $[S]_m$ designate the set of all states which have the same initial m symbols as S. In this notation, if $S \xrightarrow{I} S'$,

$$\text{pred } (S') = [S]_{K-2}. \tag{1}$$

To clarify the above notation we look at the following example, where K = 4 and the input alphabet is {0,1}.



STATE TRANSITION DIAGRAM

A solid line indicates an input of a "0" and a dashed line indicates a "1."

We can see that $011 \xrightarrow{1} 101$, 101 is the 1-successor of 011, pred (101) = {010,011} = $[011]_2$ = $[010]_2$ and $[011]_1$ = {000, 001, 010, 011}.

Extending the definition of the I-successor of a state to that of a group of states, let $I(\{S_1, \ldots, S_i\}) = \{I(S_1) \ldots, I(S_i)\}$. Therefore, if I = 1, $I([011]_2) = \{101\} = [101]_3$ and $I([011]_1) = \{100, 101\} = [101]_2$.

In general it is easy to show that if $S \xrightarrow{I} S'$ and $m \leq K-1$, we have

$$I([S]_{m-1}) = [I(S)]_m = [S']_m \tag{2}$$

$$\text{pred}\,([S']_m) = [S]_{m-1} \tag{3}$$

Equation (1) is just a special case of equation (3).

Given a state S and input symbol I, there is a corresponding code word produced by the shift register and generator polynomials (see ref 8). We will denote that code word by C(I,S).

At each state in the trellis diagram we store the metric and the survivor (the shortest path). To update the metrics, for a given state T with first symbol I, and received vector R_i, we set

$$\mu(T) = \max_{S' \epsilon \ \text{pred}(T)} \mu(S') + \log \text{prob}[R_i | C(I,S')]$$

and by choosing the state (and survivor) with the largest metric we are employing maximum likelihood decoding.

$$\text{Set } \bar{S} = [S]_{K'-1} \tag{4}$$

and

$$\mu(\bar{S}) = \max_{S' \epsilon \bar{S}} \mu(S') \tag{4'}$$

III. VARIABLE-CONSTRAINT-LENGTH CONVOLUTIONAL CODES

Up until time t_o we have a trellis diagram set up to decode a constraint length K convolutional code, at which time the constraint length suddenly reduces from K to K'. We can still decode the smaller code using the larger trellis. If we do so, we claim the following:

Proposition 1: If states S_1 and S_2 agree on the first m symbols and $m \geq K'-1$, then $C(I,S_1) = C(I,S_2)$ for all input symbols I.

Setting $m = K'-1$, we can now speak of $C(I,\bar{S})$ without ambiguity.

<u>Theorem 1</u>: If S_L is the state with the maximum metric at time t, and if the shortest path to S_L passes through S_o (at time t_o), S_1, S_2, ...,S_L, then $\mu(S_i) = \mu(\overline{S_i})$ at time t_i, for $0 \leq i \leq L$. In other words, the best path through a constraint length K trellis is equivalent to the best path through a constraint length K' trellis.

<u>Proof</u>: Clearly $\mu(S_L) = \mu(\overline{S_L})$ as S_L has the best metric of all states at t_L. Therefore, if there exists some state S_i such that $\mu(S_i) \neq \mu(\overline{S_i})$, choose i to be the last such occurrence, ie, $\mu(S_i) < \mu(\overline{S_i})$ and $\mu(S_{i+1}) = \mu(\overline{S_{i+1}})$. Suppose I is the input symbol taking S_i to S_{i+1}. Let $S_i' \epsilon \overline{S_i}$, where $\mu(S_i') = \mu(\overline{S_i})$, and let $I(S_i') = S_{i+1}'$. Then $S_{i+1}' \epsilon \overline{S_{i+1}}$ [by equation (2)] and $C(I,S_i) = C(I,S_i')$ (by Proposition 1). Therefore, the metric update factors, log prob $(R_i|C)$, are the same. But

$$\mu(S_{i+1}) = \mu(S_i) + \log \text{prob } [R_i|C(I,S_i)]$$

and
$$\mu(S_{i+1}) = \mu(S_i) + \log \text{prob } [R_i|C(I,S_i)]$$

and since $\mu(S_i) < \mu(S_i')$, we have $\mu(S_{i+1}) < \mu(S'_{i+1})$, contradicting the assumption that $\mu(S_{i+1}) = \mu(\overline{S_{i+1}})$. Therefore, no such i exists and we have proven the theorem.

Let $\mu([T]_m) = \max\limits_{S\epsilon[T]_m} \mu(S)$. [If $m = K'-1$ we would have $\mu(\bar{T})$.] for notational simplicity we will write

$$\mu(T)_m \text{ for } \mu([T]_m). \tag{5}$$

<u>Proposition 2</u>: If $I(S) = T$ and $m \geq K'1$, then

$$\mu(T)_{m+1} = \mu(S)_m + \log \text{prob } \{R|C(I,[S]_m)\}.$$

f: Set $C = C(I,[S]_m)$ (see remark following Proposition 1). Since $([T]_{m+1}) = [S]_m$ [from equation (3)], we have from the definition of $\mu(T)$:

$$\mu(T)_{m+1} = \max_{S'\varepsilon[S]_m} (\mu(S') + \log \text{prob } (R|C)).$$

n that $m \geq K'-1$, the log prob $(R|C)$ term stays constant for all $S'\varepsilon[S]_m$ Proposition 1). Therefore, $\mu(T)_{m+1} = \left(\max_{S'\varepsilon[S]_m} \mu(S')\right) + \log \text{prob } (R|C)$

$$= \mu(S)_m + \log \text{prob } (R|C)$$

llary: If $I(S) = T$ and $m \geq K'-1$, then

$$\mu(T)_m = \max_{[S']_m \subseteq [S]_{m-1}} \left\{ \mu[S']_m + \log \text{prob } \left(R|C(I,[S']_m)\right)\right\}$$

next theorem follows directly from the corollary.

rem 2: Let $K' < K$ and suppose the degrees of the generator polynomial for nstraint length K convolutional code suddenly decrease from K-1 to K'-1. ıg the Viterbi algorithm to decode the constraint length K' convolutional on a constraint length K trellis, we can simulate a constraint length K' lis by identifying all states with the same initial K'-1 bits. This racted trellis decodes in exactly the same fashion as a standard trellis a convolutional code of constraint length K'.

In light of Proposition 2, we only need to know the metrics of $[S]_m$ in r to find the next metrics of $[S]_{m+1}$. This paves the way for expanding constraint length from K' back to K.

At time t suppose that we must have a trellis diagram to accommodate a straint length K code. We assume that we have been sending a constraint th K' code from time t_o. Then at time t-1 (assuming that each information requires one time unit to process), we must know the best metrics of each he groups of states $[S]_{K-2}$. Taking the K-2 bits representing the states $_{-2}$ along with the input bit gives a state with K-1 bits for the constraint th K code. Similarly, at time t-2 we must know the best metrics of $_{-3}$. If we have been operating with a constraint length K' trellis, we

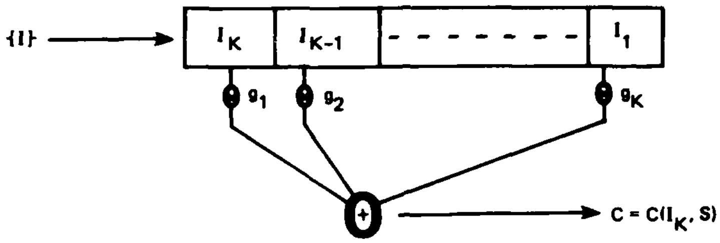
have been keeping track of only the best metrics of $[S]_{K'-1}$. Therefore, we must start expanding the trellis K-K' time units before we start decoding at constraint length K. Clearly, by contracting and expanding the trellis at the appropriate time, no information regarding the survivors or metrics will be lost.

Theorem 3: Let $K' < K$ and suppose the degrees of the generator polynomials for a constraint length K' convolutional code suddenly increase from K'-1 to K-1 at time t. Starting to expand the constraint length K' trellis (K-K') input bits before time t will ensure no loss of information on the metrics or survivors.

We should note that there will be some delay from the time the input bits arrive through the channel to the time the input bits are processed by the Viterbi algorithm. This delay, in practical applications, will be much greater than the (K-K') bit delay required by Theorem 3. If necessary, we can institute a mandatory delay in the processing of the input bits so that the bits that arrived K-K' bits previous to the change in the channel are accessible. Since the difference K-K' can never be larger than the largest possible constraint length for a given channel, a delay of M bits should be sufficient, where M is an upper bound for the largest possible constraint length.

IV. THE TIME-VARIANT HF CHANNEL

In order to discuss the complexity of decoding for the time-variant channel, we must first set up a model for the hf channel. A very simple model would consist of a shift register of length K with one generator polynomial $g(x) = \sum_{i=1}^{K} g_i x^{i-1}$, where the g_i terms are nonnegative real numbers and the input bits are $\{+1, -1\}$. The g_i terms tap the input bits in the shift register, and each bit is multiplied by the corresponding g_i and added together as in the diagram below to form the code word generated by the channel.



where $S = I_{K-1}, \ldots, I_1$.

The received signal R will be C with white Gaussian noise added. The Viterbi algorithm and state trellis diagram can be employed on our model of the hf channel.

The findings presented in the previous section dealt specifically with convolutional codes; however, all of the results can be generalized to include the channel. Equations (1), (2), and (3) hold because they deal only with the state transition diagram, which is identical for the two channels. Since the code words are generated similarly by the two channels, Proposition 1 clearly holds. It is known (ref 1) that for the hf channel, at any given time t_o, the metric at state S can be calculated from the metrics of the states mapping to S (the metrics of the predecessors of S) plus an update factor. This factor is a function of the input symbol, the states mapping to S, the code word generated from the state transition diagram, and the received vector R. If we decode a constraint length K' code on a shift register of length K, and $K'-1 \leq m \leq K-1$, by grouping the states by their initial m symbols (ie, $[S]_m$), any two states in the same group will have the same metric update factor. If we let f(I,C,R) be the metric update factor (which corresponds to log prob (R|C) for convolutional codes) then the balance of the results of section III follow, with f(I,C,R) replacing log prob (R|C).

V. CATASTROPHIC CODES

Inasmuch as we have no control over the intersymbol interference pattern of the channel, it is possible that nature will provide us with a poor code. In fact, we will give a class of codes which is catastrophic. A code is, in

general, said to be catastrophic if a finite number of channel errors (or noise) can cause an infinite number of decoded bit errors. This is the result of two input sequences differing in an infinite number of positions that generate the same output sequence of code words except for a finite number of initial discrepancies. We first examine codes in which the nonzero taps are equal in number.

For example, consider the channel where $K = 2$ and $g_1 = g_2$. The two input sequences

$$X = +1-1+1-1\ \ldots \tag{6}$$

and

$$Y = -1+1-1+1\ \ldots$$

will produce an output sequence of all zeros once the second input bit of each sequence is inserted into the shift register.

We have just shown that the $K = 2$, $g_1 = g_2$ code is catastrophic. We will show that any code with equally spaced nonzero equal taps is catastrophic.

A collection of m input sequences will be called a <u>catastrophic m-tuple</u> if the individual encoded output sequences are all the same except for a finite number of initial discrepancies. The existence of these sequences induces a catastrophic code. Given two input sequences

$$A = a_1 a_2 \ldots \qquad \text{and} \qquad B = b_1 b_2 \ldots,$$

we can create a new sequence $A^*B = a_1 b_1 a_2 b_2 \ldots.$

Note that the operation * is neither commutative nor associative. To interleave the bits from three sequences A,B, and C, we write $(A^*B^*C)_3 = a_1 b_1 c_1 a_2 b_2 c_2 \ldots.$ [Note that this is <u>not</u> $(A^*B)^*C$.] The generalization to n sequences should be clear.

If we have two nonzero taps $g_1 = g_3$, the following sequences are a catastrophic quadruple: $X*X$, $X*Y$, $Y*X$ and $Y*Y$, where X and Y are alternating sequences of ±1 from equation (6). All four input sequences eventually yield the all zero output sequence.

For constraint length K and two nonzero taps $g_1 = g_K$, we have a catastrophic 2^{K-1}-tuple consisting of all possible ways of interleaving K-1 X's and Y's together. In fact, if H_i, $1 \le i \le K-2$, are any input sequences, then

$$(X*H_1*\cdots*H_{K-2})_{K-1}$$

and

$$(Y*H_1*\cdots*H_{K-2})_{K-1}$$

form a catastrophic pair

Next we consider the code which taps every bit equally, ie, $g_1 = g_2 = \cdots = g_k > 0$. Let P be the sequence of all plus 1's and M the sequence of all minus 1's. Then a catastrophic (K-1)-tuple would be $S_1 = (M*P*P*\cdots*P)_{K-1}$, $S_2 = (P*M*P*\cdots*P)_{K-1}$, ..., $S_{K-1} = (P*P*P*\cdots P*M)_{K-1}$, all eventually producing the same output sequence. By interchanging P and M above, we have another catastrophic (K-1)-tuple.

Finally, for equally spaced nonzero equal taps, we set $g_1 > 0$ and the remaining taps will either be zero or equal to g_1. We can describe this by

$$g_i = \begin{cases} g_1 & \text{if } i \equiv 1 \pmod t \\ 0 & \text{otherwise} . \end{cases}$$

If the code is to be of constraint length K, then $K \equiv 1 \pmod t$. We state the following proposition, the proof of which is immediate.

Proposition 3: Let C_R be a constraint length R code with $g_1 = g_2 \cdots = g_R > 0$, and let C_K be a constraint length K code with every t^{th} bit tapped starting with g_1, where $K = (R-1)t+1$. Then if we input the t sequences $A_1, \ldots, A_t$ into

to generate the t output sequences $0_1,\ldots,0_t$, respectively, and if we input $\left(A_1*A_2*\ldots*A_t\right)_t$ into C_K, we get the output sequence $\left(0_1*\ldots*0_t\right)_t$.

We have shown the input sequences $S_1 = (M*P*P*\ldots*P)_{R-1},\ldots, S_{R-1} = (P*P*\ldots*M)_{R-1}$ are a catastrophic (R-1)-tuple for the code C_R. Therefore, if we interleave any t of the sequences $S_1,\ldots, S_{R-1}$ together, it follows directly from Proposition 3 that they will form a catastrophic $(R-1)^t$-tuple for C_K.

We give one final look at catastrophic codes with unequal taps. Consider the code in which $g_1 = g_3$ and $g_2 = 2g_1$. The input sequences X and Y are a catastrophic pair. If differential encoding is used, however, the sequences X and Y become the same sequence, which saves this code from being catastrophic.

Another example of a catastrophic code with unequal taps is provided by $g_1 = g_3$, and $g_2 = g_4 = 2g_1$. The sequences $(P*P*M*M)_4$, and $(M*P*P*M)_4$ form a catastrophic 4-tuple, and these input sequences are distinct, even if differential encoding is employed.

VI. CONCLUSIONS

For multipath channels with intersymbol interference, there exists a modified Viterbi algorithm that is maximum likelihood. For time-variant, fading channels where the constraint length may expand or contract due to the latest arriving multipath, given that the channel is known at all times, we have shown that the complexity of the algorithm depends upon the constraint length at which it is currently decoding. The same holds true if the channel encoder were instead a standard convolutional code with the Viterbi decoding algorithm. If the earliest received signal fades in and out, it is conjectured that the same reduction in the complexity of the algorithm can be shown by identifying states by their last m bits and building an input delay before the bits are inserted into the shift register.

The intersymbol interference due to the hf multipath channel may encode in a fashion that causes the channel to become catastrophic. This happens

when the echoes occur at regular intervals and are all of equal strength and, in some cases, where the taps are unequal in magnitude.

An interesting topic for further study would be the investigation of the error rate produced by a catastrophic code. When the code goes into a catastrophic burst of errors, in some instances the actual bit error rate is very low. For the case in which $g_1 = g_2 = \ldots = g_K$, any two sequences from the given catastrophic (K-1)-tuple will produce an error rate of $\frac{2}{K-1}$ during that burst. Although the performance of the code may decrease because of its catastrophic nature, it does not necessarily mean that the code is impractical.

Another topic, not touched upon in this paper, is the probability that the channel will give a catastrophic code. As the taps must be exactly in a certain ratio and the path spacing must be exact multiple of the baud rate to produce a catastrophic code, the percentage of time that we are decoding a catastrophic code will be small. These problems should be explored further.

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